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Spatial confinement effects on a quantum harmonic oscillator: nonlinear coherent state approach

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Abstract
In this paper we study some basic quantum confinement effects through investigation of a deformed harmonic oscillator algebra. We show that spatial confinement effects on a quantum harmonic oscillator can be represented by a deformation function within the framework of nonlinear coherent states theory. We construct the coherent states associated with the spatially confined quantum harmonic oscillator in a one-dimensional infinite well and examine some of their quantum statistical properties, including sub-Poissonian statistics and quadrature squeezing.

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1. Introduction

The harmonic oscillator is one of the models most extensively used in both classical and quantum mechanics. The usefulness and simplicity make this model a subject of lots of studies. One of the most important aspects of the quantum harmonic oscillator (QHO) is its dynamic algebra, i.e. Weyl–Heisenberg algebra. This algebra appears in many areas of modern theoretical physics, as an example we note that the one-dimensional quantum harmonic oscillator was successfully used in second quantization formalism [1].

Due to the relevance of Weyl–Heisenberg algebra, some efforts have been devoted to the study of possible deformations of the QHO algebra [2]. A deformed algebra is a nontrivial generalization of a given algebra through the introduction of one or more deformation parameters, such that, in a certain limit of the parameters the non-deformed algebra is recovered. A particular deformation of the Heisenberg algebra has led to the notion of f-oscillator [3]. An f-oscillator is a non-harmonic system, that from mathematical point of view its dynamical variables (creation and annihilation operators) are constructed from a non-canonical transformation through

$$\hat{A} = \hat{a} f(\hat{n}), \qquad \hat{A}^{\dagger} = f(\hat{n})\hat{a}^{\dagger}, \tag{1}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the usual (non-deformed) harmonic oscillator operators with $[\hat{a}, \hat{a}^\dagger] = 1$ and $\hat{n} = \hat{a}^\dagger\hat{a}$. The function $f(\hat{n})$ is called deformation function which depends on the number of excitation quanta and some physical parameters. The presence of the operator-valued deformation function causes the Heisenberg algebra of the standard QHO to transform into a deformed Heisenberg algebra. The nonlinearity in f-oscillators means dependence of the oscillation frequency on the intensity [4]. On the other hand, in contrast to the standard QHO, f-oscillators do not have an equal-spaced energy spectrum. For example, if we confine a simple QHO inside an infinite well, due to the spatial confinement, the energy levels constitute a spectrum that is not equal spaced. Therefore, in this case it is reasonable to investigate the corresponding f-oscillator.

The confined QHO can be used to describe confinement effects on physical properties of confined systems. Physical size and shape of the materials strongly affect the nature, dynamics of the electronic excitations, lattice vibrations and dynamics of carriers. For example, in the mesoscopic systems, the dimension of system is comparable with the coherence length of carriers and this leads to some new phenomena that they do not appear in a bulk semiconductor, such as quantum interference between carrier's motion [5]. Recent progress in growth techniques and development of micromachining technology in designing mesoscopic systems and nanostructures have led to intensive theoretical [6] and experimental investigations [7] on electronic and optical properties of those systems. The most important point about the nanoscale structures is that the quantum confinement effects play the center-stone role. One can even say, in general, that recent success in the nanofabrication technique has resulted in great interest in various artificial physical systems (quantum dots, quantum wires and quantum wells) with new phenomena driven by the quantum confinement. A number of recent experiments have demonstrated that isolated semiconductor quantum dots are capable of emitting light [8]. It becomes possible to combine high-Q optical microcavities with quantum dot emitters as the active medium [9]. Furthermore, there are many theoretical attempts for understanding the optical and electronic properties of nanostructures especially semiconductor quantum dots [10]. On the other hand, a nanostructure such as quantum dot is a system that the carrier's motion is confined inside a small region, and during the interaction with other systems, the generated excitations such as phonons, excitons and plasmons are confined in a small region. In order to describe the physical properties of these excitations one can consider them as harmonic oscillators.

As another application of deformed algebra we can refer to the notion of parastatistics [11]. The concept of parastatistics has found many applications in fractal statistics and anyon theory [12]. In addition to the anyon theory, the parastatistics has found many interesting applications in supersymmetry and non-commutative quantum mechanics [13].

The construction of generalized deformed oscillators corresponding to well-known potentials and study of the correspondence between the properties of the conventional potential picture and the algebraic one has been done [14]. Recently, the generalized deformed algebra and its associated generalized operators have been considered [15]. By looking at the classical correspondence of the Hamiltonian, the potential energy and the effective mass function are obtained. In this contribution, we derive the generalized operators associated with a definite potential by comparing the physical properties of system and physical results of generalized algebra.

One of the most interesting features of the QHO is the construction of its coherent states as the eigenfunctions of the annihilation operator. As is well known [3], one can introduce nonlinear coherent states (NLCSs) or f-coherent states as the right-hand eigenstates of the deformed annihilation operator $\hat{A}$. It has been shown [16] that these families of generalized coherent states exhibit various non-classical properties. Due to these properties and their

applications, generation of these states is a very important issue in the context of quantum optics. The f-coherent states may appear as stationary states of the center-of-mass motion of a trapped ion [17]. Furthermore, a theoretical scheme for generation of these states in a coherently pumped micromaser within the framework of intensity-dependent Jaynes–Cummings model has been proposed [18]. Recently, it was shown that it is possible to construct a special family of NLCSs in the stationary state of an excitor inside a wide quantum dot [20].

One of the most important questions is the physical meaning of the deformation in the NLCSs theory. It has been shown [20] that there is a close connection between the deformation function that appears in the algebraic structure of NLCSs and the non-commutative geometry of the configuration space. Furthermore, it has been shown recently [21] that a two-mode QHO confined on the surface of a sphere, can be interpreted as a single-mode deformed oscillator, whose quantum statistics depends on the curvature of sphere.

Motivated by the above-mentioned studies, in the present contribution we intend to investigate the spatial confinement effects on physical properties of a standard QHO. It will be shown that the spatial confinement leads to deformation of a standard QHO. We consider a QHO confined in a one-dimensional infinite well without periodic boundary conditions, and we find its energy levels, as well as associated ladder operators. We show that the ladder operators can be interpreted as a special kind of the so-called f-deformed creation and annihilation operators [3].

This paper is organized as follows: in section 2, we review some physical properties of the f-oscillator and its coherent states. In section 3, we consider the spatially confined QHO in a one-dimensional infinite well and construct its associated coherent states. We shall also examine some of their quantum statistical properties, including sub-Poissonian statistics and quadrature squeezing. Finally, we summarize our conclusions in section 4.

2. f-oscillator and nonlinear coherent states

In this section, we review the basics of the f-deformed quantum oscillator and the associated coherent states known in the literature as nonlinear coherent states. At first, to investigate one of the sources of deformation we consider an eigenvalue problem for a given quantum physical system and we focus our attention on the properties of creation and annihilation operators that allow us to make transition between the states of discrete spectrum of the system Hamiltonian [22]. As usual, we expand the Hamiltonian in its eigenvectors

$$\hat{H} = \sum_{i=0}^{\infty} E_i |i\rangle\langle i|, \tag{2}$$

where we have chosen $E_0 = 0$. We introduce the creation (raising) and annihilation (lowering) operators as follows:

$$\hat{a}^\dagger = \sum_{i=0}^{\infty} \sqrt{E_{i+1}} |i+1\rangle\langle i|, \qquad \hat{a} = \sum_{i=0}^{\infty} \sqrt{E_i} |i-1\rangle\langle i|, \tag{3}$$

so that $\hat{a}|0\rangle = 0$. These ladder operators satisfy the following commutation relation:

$$[\hat{a}, \hat{a}^\dagger] = \sum_{i=1}^{\infty} (E_{i+1} - E_i) |i\rangle\langle i|. \tag{4}$$

Obviously if the energy spectrum is equally spaced that is, it should be linear in quantum numbers, as in the case of ordinary QHO, then $E_{i+1} - E_i = c$, where c is a constant and in

this condition the commutator of $\hat{a}$ and $\hat{a}^\dagger$ becomes a constant (a rescaled Weyl–Heisenberg algebra). On the other hand, if the energy spectrum is not equally spaced, the ladder operators of the system satisfy a deformed Heisenberg algebra, i.e. their commutator depends on the quantum numbers that appear in the energy spectrum. This is one of the most important properties of the quantum f-oscillators [3].

An f-oscillator is a non-harmonic system characterized by a Hamiltonian of the harmonic oscillator form

$$\hat{H}_D = \frac{\Omega}{2}(\hat{A}\hat{A}^\dagger + \hat{A}^\dagger\hat{A}) \qquad (\hbar = 1), \tag{5}$$

($\hat{A} = \hat{a}f(\hat{n})$) with a specific frequency Ω and deformed boson creation and annihilation operators defined in (1). The deformed operators obey the commutation relation

$$[\hat{A}, \hat{A}^\dagger] = (\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n}). \tag{6}$$

The f-deformed Hamiltonian $\hat{H}_D$ is diagonal on the eigenstates $|n\rangle$ in the Fock space and its eigenvalues are

$$E_n = \frac{\Omega}{2}[(n+1)f^2(n+1) + nf^2(n)]. \tag{7}$$

In the limit $f \to 1$, the ordinary expression $E_n = \Omega\left(n + \frac{1}{2}\right)$ and the usual (non-deformed) commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$ are recovered.

Furthermore, by using the Heisenberg equation of motion with Hamiltonian (5)

$$\mathrm{i}\frac{\mathrm{d}\hat{A}}{\mathrm{d}t} = [\hat{A}, \hat{H}_D], \tag{8}$$

we obtain the following solution for the f-deformed operators $\hat{A}$ and $\hat{A}^\dagger$:

$$\hat{A}(t) = \mathrm{e}^{-\mathrm{i}\Omega G(\hat{n})t}\hat{A}(0), \qquad \hat{A}^\dagger(t) = \hat{A}^\dagger(0)\,\mathrm{e}^{\mathrm{i}\Omega G(\hat{n})t}, \tag{9}$$

where

$$G(\hat{n}) = \tfrac{1}{2}((\hat{n}+2)f^2(\hat{n}+2) - \hat{n}f^2(\hat{n})). \tag{10}$$

In this sense, the f-deformed oscillator can be interpreted as a nonlinear oscillator whose frequency of vibrations depends explicitly on its number of excitation quanta [4]. It is interesting to point out that recent studies have revealed strictly physical relationship between the nonlinearity concept resulting from the f-deformation and some nonlinear optical effects, e.g., Kerr nonlinearity, in the context of atom–field interaction [23].

The nonlinear transformation of the creation and annihilation operators leads naturally to the notion of nonlinear coherent states or f-coherent states. The nonlinear coherent states $|\alpha\rangle_f$ are defined as the right-hand eigenstates of the deformed operator

$$\hat{A}|\alpha\rangle_f = \alpha|\alpha\rangle_f. \tag{11}$$

From equation (11) one can obtain an explicit form of the nonlinear coherent states in a number state representation

$$|\alpha\rangle_f = C\sum_{n=0}^{\infty}\alpha^n d_n|n\rangle, \tag{12}$$

where the coefficients d_n's and normalization constant C are, respectively, given by

$$d_0 = 1, \qquad d_n = (\sqrt{n!}[f(n)]!)^{-1}, \qquad [f(n)]! = \prod_{j=1}^{n} f(j), \qquad C = \left(\sum_{n=0}^{\infty} d_n^2|\alpha|^{2n}\right)^{-\frac{1}{2}}. \tag{13}$$

In recent years the nonlinear coherent states have been paid much attentions because they exhibit nonclassical features [16] and many quantum optical states, such as squeezed states, phase states, negative binomial states and photon-added coherent states can be viewed as a sort of nonlinear coherent states [24].

3. A quantum harmonic oscillator in a one-dimensional infinite well

3.1. The f-deformed oscillator description of a confined QHO

In this section we consider a quantum harmonic oscillator confined in a one-dimensional infinite well. Many attempts have been made to solve this problem (see [25, 26], and references therein). In most of those works, the authors tried to solve the problem numerically. But in our consideration we try to solve the problem analytically, to reveal the relationship between the confinement effect and given deformation function. We start from the Schrödinger equation ($\hbar = 1$)

$$\left[-\frac{1}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}kx^2 + V(x)\right]\psi(x) = E\psi(x), \tag{14}$$

where

$$V(x) = \begin{cases} 0 & -a \leqslant x \leqslant a \\ \infty & \text{elsewhere.} \end{cases}$$

According to the approach introduced in the previous section, we can obtain raising and lowering operators from the spectrum of the Schrödinger operator. On the other hand, by comparing the energy spectrum of a particular system with the energy spectrum of the general f-deformed oscillator (7), one could obtain deformed raising and lowering operators. Hence, we need an analytical expression for energy spectrum of the system which explicitly shows dependence on special quantum numbers. The original problem, a confined QHO (14), can be solved only by using the approximation methods. When applying perturbation theory, one is usually concerned with a small perturbation of an exactly solvable Hamiltonian system. In the case of the confined QHO we deal with three limits. Inside the well, for small values of position we have a harmonic oscillator, for large values we have an infinite well and at the positions of the boundaries the two potentials have the same power. Hence the approximation method cannot lead to acceptable results. Therefore, we model the original problem by a model potential that has mathematical behavior such as a confined QHO. Instead of solving the Schrödinger equation for the QHO confined between infinite rectangular walls in positions $\pm a$, we propose to solve the eigenvalue problem for the potential

$$V(x) = \frac{1}{2}k\left(\frac{\tan(\delta x)}{\delta}\right)^2, \tag{15}$$

where $\delta = \frac{\pi}{2a}$ is a scaling factor depending on the width of the well. This potential models a QHO placed in the center of the rectangular infinite well [27]. The potential $V(x)$ (15) fulfils two asymptotic requirements: (1) $V(x) \to \frac{1}{2}kx^2$ when $a \to \infty$ (free harmonic oscillator limit). (2) $V(x)$ at equilibrium position has the same curvature as a free QHO, $\left[\frac{\mathrm{d}^2V}{\mathrm{d}x^2}\right]_{x=0} = k$. This model potential belongs to the exactly solvable trigonometric Pöschl–Teller potentials family [28]. Stationary coherent states for a special kind of this potential have been considered [29].

Now we consider the following equation:

$$\left[-\frac{1}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}k\left(\frac{\tan(\delta x)}{\delta}\right)^2 - E\right]\psi(x) = 0. \tag{16}$$

Table 1. Calculated energy levels of the confined QHO in a one-dimensional infinite well by using our model potential in comparison with the numerical results given in [25].

State	Boundary size	Model potential	Numerical results
0	$a = 0.5$	4.984 953 12	4.951 129 32
0	1	1.410 893 25	1.298 459 83
0	2	0.677 453 92	0.537 461 20
0	3	0.573 214 64	0.500 391 08
0	4	0.540 037 28	0.500 000 49
1	$a = 0.5$	19.889 661 57	19.774 534 17
1	1	5.466 380 33	5.075 582 01
1	2	2.340 786 91	1.764 816 43
1	3	1.856 721 76	1.506 081 52
1	4	1.697 218 13	1.500 014 61
2	$a = 0.5$	44.663 974 41	44.452 073 82
2	1	11.989 268 50	11.258 825 78
2	2	4.620 970 17	3.399 788 24
2	3	3.414 384 55	2.541 127 25
2	4	3.008 611 55	2.500 201 17
3	$a = 0.5$	79.307 891 66	78.996 921 15
3	1	20.979 557 77	19.899 696 49
3	2	7.518 003 71	5.584 639 07
3	3	5.246 203 03	3.664 219 64
3	4	4.474 217 54	3.501 691 53
4	$a = 0.5$	123.821 413 30	123.410 710 50
4	1	32.437 248 14	31.005 254 50
4	2	11.031 887 52	8.368 874 42
4	3	7.352 177 18	4.954 180 47
4	4	6.094 036 10	4.509 640 99

To solve this equation analytically, we use the factorization method [30]. By changing the variable and some mathematical manipulation, the corresponding energy eigenvalues are found as

$$E_n = \gamma \left(n + \tfrac{1}{2}\right)^2 + \sqrt{\gamma^2 + \omega^2}\left(n + \tfrac{1}{2}\right) + \frac{\gamma}{4}, \tag{17}$$

where $\gamma = \frac{4\pi^2}{32a^2m}$ and $\omega = \sqrt{\frac{k}{m}}$ is the frequency of the QHO. The first term in the energy spectrum can be interpreted as the energy of a free particle in a well, the second term denotes the energy spectrum of the QHO, and the last term shifts the energy spectrum by a constant amount. It is evident that if $a \to \infty$ then $\gamma \to 0$ and the energy spectrum (17) reduces to the spectrum of a free QHO. As is clear from (17), different energy levels are not equally spaced. Hence, confining a free QHO leads to deformation of its dynamical algebra and we can interpret the parameter γ as the corresponding deformation parameter. In table 1 the numerical results associated with the original potential, given in [25], are compared with the generated results from the model potential under consideration. As is seen, the results are in good agreement when the boundary size is of the order of characteristic length of the harmonic oscillator. The original oscillator potential becomes infinite suddenly when it approaches the boundaries of the well, while the model potential is smooth and approaches to the infinity asymptotically. Therefore, the model potential (15) is more appropriate for the physical systems.

If we normalize equation (17) to the energy quanta of the simple harmonic oscillator and introduce the new variables $n + \frac{1}{2} = h$, $\sqrt{\frac{\gamma^2}{\omega^2} + 1} = \eta$ and $\gamma' = \frac{\gamma}{\omega}$ then it takes the following form:

$$E_l = \gamma' h^2 + \eta h + \frac{\gamma'}{4}. \tag{18}$$

By comparing this spectrum with the energy spectrum of an f-deformed oscillator, given by (7), we find the corresponding deformation function as

$$f(\hat{n}) = \sqrt{\gamma' \hat{n} + \eta}. \tag{19}$$

Furthermore, the ladder operators associated with the confined oscillator under consideration can be written in terms of the conventional (non-deformed) operators $\hat{a}$, $\hat{a}^\dagger$ as follows:

$$\hat{A} = \hat{a}\sqrt{\gamma' \hat{n} + \eta}, \qquad \hat{A}^\dagger = \sqrt{\gamma' \hat{n} + \eta}\hat{a}^\dagger. \tag{20}$$

These two operators satisfy the following commutation relation:

$$[\hat{A}, \hat{A}^\dagger] = \gamma'(2\hat{n} + 1) + \eta. \tag{21}$$

It is obvious that in the limiting case $a \to \infty$ ($\gamma' \to 0, \eta \to 1$), the right-hand side of the above commutation relation becomes independent of $\hat{n}$, and the deformed algebra reduces to a conventional Weyl–Heisenberg algebra for a free QHO.

Classically, a harmonic oscillator is a particle that is attached to an ideal spring and can oscillate with specific amplitude. When that particle is confined, boundaries can affect the particle's motion if the boundary position is a smaller distance in comparison with the characteristic length that the particle oscillates within. This characteristic length for the QHO is given by $\frac{\hbar}{m\omega}(\hbar = 1)$, and if $2a \leqslant \frac{1}{m\omega}$, then the presence of the boundaries affects the behavior of the QHO, otherwise it behaves like a free QHO. Therefore, one can interpret $l_0 = \frac{1}{m\omega}$ as a scale length where the deformation effects become relevant.

3.2. Coherent states of confined oscillator

Now, we focus our attention on the coherent states associated with the QHO under consideration. As usual, we define the coherent states as the right-hand eigenstates of the deformed annihilation operator

$$\hat{A}|\beta\rangle_f = \beta|\beta\rangle_f. \tag{22}$$

From (22) and using the NLCS formalism introduced in (11)–(13) the explicit form of the corresponding NLCS of the confined QHO is written as

$$|\beta\rangle_f = \mathcal{N} \sum_n \frac{\beta^n}{\sqrt{n!(\gamma' n + \eta)!}} |n\rangle, \tag{23}$$

where $\mathcal{N} = \left(\sum_n \frac{|\beta|^{2n}}{[f(n)!]^2 n!}\right)^{-\frac{1}{2}}$ is the normalization factor, β is a complex number and the deformation function $f(n)$ is given by equation (19). The ensemble of states $|\beta\rangle_f$ labelled by the single complex number β is called a set of coherent states if the following conditions are satisfied [31]:

- normalizability

$$_f\langle\beta|\beta\rangle_f = 1, \tag{24}$$

- continuity in the label β

$$|\beta - \beta'| \to 0 \quad \Rightarrow \quad \||\beta\rangle_f - |\beta'\rangle_f\| \to 0, \tag{25}$$

- resolution of the identity

$$\int_c \mathrm{d}^2\beta|\beta\rangle_{f\,f}\langle\beta|w(|\beta|^2) = \hat{I}, \tag{26}$$

where $w(|\beta|^2)$ is a proper measure that ensures the completeness and the integration is restricted to the part of the complex plane where normalization converges.

The first two conditions can be proved easily. For the third condition, we choose the normalization constant as

$$\mathcal{N}^2 = \frac{|\beta|^\eta}{I_\eta^{\gamma'}(2|\beta|)}, \tag{27}$$

where

$$I_\eta^{\gamma'}(x) = \sum_{s=0}^{\infty} \frac{1}{s!(\gamma' s+\eta)!}\left(\frac{x}{2}\right)^{2s+\eta} \tag{28}$$

is similar to the modified Bessel function of the first kind of the order η with the series expansion $I_\eta(x) = \sum_{s=0}^{\infty} \frac{1}{s!(s+\eta)!}\left(\frac{x}{2}\right)^{2s+\eta}$. Resolution of the identity of the deformed coherent states $|\beta\rangle_f$ can be written as

$$\int \mathrm{d}^2\beta|\beta\rangle_f w(|\beta|)_f\langle\beta| = \pi\sum_n |n\rangle\langle n|\frac{1}{n!(\gamma' n+\eta)!}\int_0^\infty \mathrm{d}|\beta||\beta||\beta|^{2n}\frac{|\beta|^\eta}{I_\eta^{\gamma'}(2|\beta|)}w(|\beta|). \tag{29}$$

Now we introduce the new variable $|\beta|^2 = x$ and the measure

$$w(\sqrt{x}) = \frac{8}{\pi}I_\eta^{\gamma'}(2\sqrt{x})K_m(2\sqrt{x})x^{\frac{l}{2}}, \tag{30}$$

where $K_m(x)$ is the modified Bessel function of the second kind of the order m, $m = (\gamma'-1)n+\alpha$, and $l = (\gamma'-1)n+1$. Using the integral relation $\int_0^\infty K_\nu(t)t^{\mu-1}\,\mathrm{d}t = 2^{\mu-2}\Gamma\left(\frac{\mu-\nu}{2}\right)\Gamma\left(\frac{\mu+\nu}{2}\right)$ [32], we obtain

$$\int \mathrm{d}^2\beta|\beta\rangle_f w(|\beta|)_f\langle\beta| = \sum_n |n\rangle\langle n| = \hat{I}. \tag{31}$$

We therefore conclude that the states $|\beta\rangle_f$ qualify as coherent states in the sense described by conditions (24)–(26).

We now proceed to examine some nonclassical properties of the nonlinear coherent states $|\beta\rangle_f$. As an important quantity, we consider the variance of the number operator $\hat{n}$. Since for the coherent states the variance of the number operator is equal to its average, deviation from the Poissonian statistics can be measured with the Mandel parameter [33]

$$M = \frac{(\Delta n)^2 - \langle\hat{n}\rangle}{\langle\hat{n}\rangle}. \tag{32}$$

This parameter vanishes for the Poisson distribution, is positive for the super-Poissonian distribution (bunching effect) and is negative for the sub-Poissonian distribution (antibunchig effect). Figure 1 shows the size dependence of the Mandel parameter for different values of $|\beta|^2$. As is seen, the Mandel parameter exhibits the sub-Poissonian statistics and with further increasing values of a it is finally stabilized at an asymptotical zero value corresponding to the Poissonian statistics. In addition, the smaller the parameter $|\beta|^2$ is, the more rapidly the Mandel parameter tends to the Poissonian statistics.

As another important nonclassical property we examine the quadrature squeezing. For this purpose we first consider the conventional quadrature operators $\hat{X}_a$ and $\hat{Y}_a$ defined in terms of nondeformed operators $\hat{a}$ and $\hat{a}^\dagger$ as [34]

$$\hat{X}_a = \frac{1}{2}(\hat{a}\,\mathrm{e}^{\mathrm{i}\phi} + \hat{a}^\dagger\,\mathrm{e}^{-\mathrm{i}\phi}), \qquad \hat{Y}_a = \frac{1}{2\mathrm{i}}(\hat{a}\,\mathrm{e}^{\mathrm{i}\phi} - \hat{a}^\dagger\,\mathrm{e}^{-\mathrm{i}\phi}). \tag{33}$$

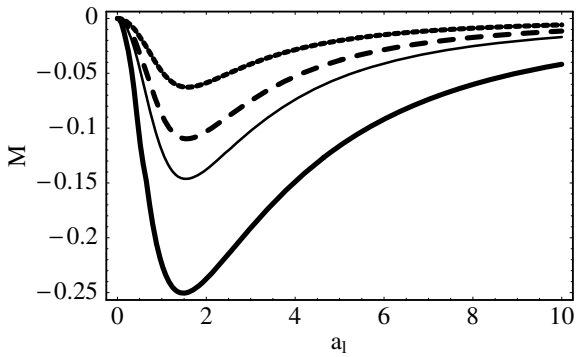

Figure 1. Plots of the Mandel parameter versus the dimensionless parameter $a_l = \frac{a}{l_0}$. For $|\beta|^2 = 0.5$ (dashed curve), for $|\beta|^2 = 1$ (longdashed curve), for $|\beta|^2 = 1.5$ (solid curve) and for $|\beta|^2 = 4.0$ (bold curve).

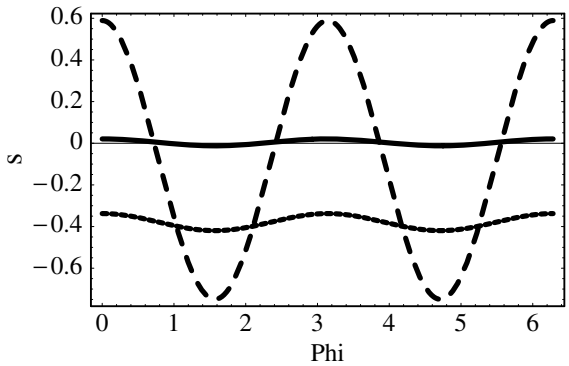

Figure 2. Plot of $s_{\hat{X}_a}$ versus ϕ for $|\beta|^2 = 4$. The dashed, longdashed and solid curves respectively relate to $a = 2.5$, $a = 1$, $a = 0.5$ (the values of a are renormalized to l_0).

In this equation, ϕ is the phase of quadrature operators which can effectively affect the squeezing properties. The commutation relation for $\hat{a}$ and $\hat{a}^\dagger$ leads to the following uncertainty relation:

$$(\Delta\hat{X}_a)^2(\Delta\hat{Y}_a)^2 \geqslant \tfrac{1}{4}|\langle[\hat{X}_a, \hat{Y}_a]\rangle|^2 = \tfrac{1}{16}. \tag{34}$$

For the vacuum state $|0\rangle$, we have $(\Delta\hat{X}_a)^2 = (\Delta\hat{Y}_a)^2 = \frac{1}{4}$ and hence $(\Delta\hat{X}_a)^2(\Delta\hat{Y}_a)^2 = \frac{1}{16}$. A given quantum state of the QHO is said to be squeezed when the variance of one of the quadrature components $\hat{X}_a$ and $\hat{Y}_a$ satisfies the relation

$$(\Delta\hat{O}_a)^2 < (\Delta\hat{O}_a)^2_{\text{vacuum}} = \tfrac{1}{4} \quad (\hat{O}_a = \hat{X}_a \text{ or } \hat{Y}_a). \tag{35}$$

The degree of quadrature squeezing can be measured by the squeezing parameter $s_{\hat{O}}$ defined by

$$s_{\hat{O}} = 4(\Delta\hat{O}_a)^2 - 1. \tag{36}$$

Then, the condition for squeezing in the quadrature component can be simply written as $s_{\hat{O}} < 0$. In figure 2 we have plotted the parameter $s_{\hat{X}_a}$ corresponding to the squeezing of $\hat{X}_a$ with respect to the phase angle ϕ for four different values of a. As is seen, the state $|\beta\rangle_f$ exhibits squeezing for different values of the confinement size, and when $a_l = \frac{a}{l_0} = 2.5$, the

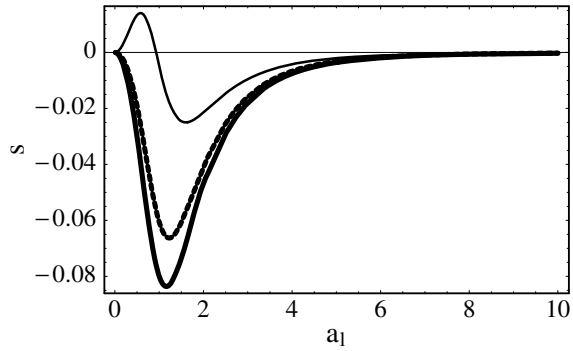

Figure 3. Plots of $s_{\hat{X}_a}$ versus the dimensionless parameter $a_l = \frac{a}{l_0}$ for different phases and $|\beta|^2 = 1$. Dashed curve, solid curve and bold curve, respectively, correspond to $\phi = 100$, $\phi = 110$ and $\phi = 90$.

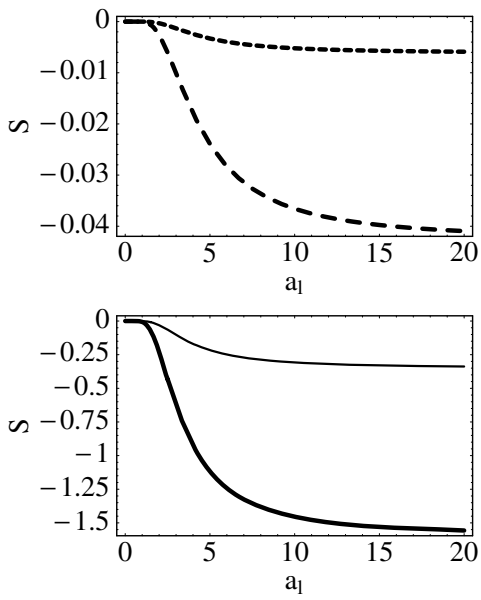

Figure 4. Plots of deformed squeezing parameter S_{X_A} versus the dimensionless parameter $a_l = \frac{a}{l_0}$. The dashed curve, longdashed curve, solid curve and bold curve respectively, correspond to $|\beta|^2 = 1$, $|\beta|^2 = 1.5$, $|\beta|^2 = 2.5$ and $|\beta|^2 = 4$.

quadrature $\hat{X}_a$ exhibits squeezing for all values of the phase angle ϕ. Figure 3 shows the plot of $s_{\hat{X}_a}$ versus the dimensionless parameter $a_l = \frac{a}{l_0}$ for different values of the phase ϕ. As is seen, with the increasing value of $a_l\left(\frac{a}{l_0}\right)$, the quadrature component tends to zero according to the vacuum fluctuation. Let us also consider the deformed quadrature operators $\hat{X}_A$ and $\hat{Y}_A$ defined in terms of the deformed operators $\hat{A}$ and $\hat{A}^\dagger$ as

$$\hat{X}_A = \frac{1}{2}(\hat{A}\,\mathrm{e}^{\mathrm{i}\phi} + \hat{A}^\dagger\,\mathrm{e}^{-\mathrm{i}\phi}), \qquad \hat{Y}_A = \frac{1}{2\mathrm{i}}(\hat{A}\,\mathrm{e}^{\mathrm{i}\phi} - \hat{A}^\dagger\,\mathrm{e}^{-\mathrm{i}\phi}). \tag{37}$$

By considering the commutation relation (6) for the deformed operators $\hat{A}$ and $\hat{A}^\dagger$, the squeezing condition for the deformed quadrature operators $\hat{O}_A$ $(= \hat{X}_A, \hat{Y}_A)$ can be written as

$$S = 4(\Delta \hat{O}_A)^2 - \langle (\hat{n}+1) f^2(\hat{n}+1) \rangle + \langle \hat{n} f^2(\hat{n}) \rangle < 0. \tag{38}$$

In figure 4 we have plotted the parameter $S_{\hat{X}_A}$ versus the dimensionless parameter $\frac{a}{l_0}$ for four different values of $|\beta|^2$. As is seen, the deformed quadrature operator exhibits squeezing for all values of a. Furthermore, with the increasing value of $|\beta|^2$ the squeezing of the quadrature $\hat{X}_A$ is enhanced.

4. Conclusion

In this paper, we have considered the relation between the spatial confinement effects and a special kind of f-deformed algebra. We have found that the confined simple harmonic oscillator can be interpreted as an f-oscillator, and we have obtained the corresponding deformation function. By constructing the associated NLCSs, we have examined the effects of the confinement size on non-classical statistical properties of those states. The result shows that the stronger confinement leads to the strengthening of non-classical properties. We hope that our approach may be used in the description of phonons in the strong excitation regimes, photons in a microcavity and different elementary excitations in confined systems. The work in this direction is in progress.

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